

Giải bài tập Toán 10 SBT bài 3 chương 6

Bài 16 trang 193 Sách bài tập (SBT) Toán Đại số 10

Cho $\cos \alpha = \frac{1}{3}$ tính $\sin(\alpha + \frac{\pi}{6}) - \cos(\alpha - \frac{2\pi}{3})$

Gợi ý làm bài

Ta có:

$$\begin{aligned} & \sin(\alpha + \frac{\pi}{6}) - \cos(\alpha - \frac{2\pi}{3}) \\ &= \sin \alpha \cos \frac{\pi}{6} + \cos \alpha \sin \frac{\pi}{6} - \cos \alpha \cos \frac{2\pi}{3} - \sin \alpha \sin \frac{2\pi}{3} \\ &= \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha + \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha \\ &= \cos \alpha = \frac{1}{3} \end{aligned}$$

Bài 17 trang 193 Sách bài tập (SBT) Toán Đại số 10

Cho $\sin \alpha = \frac{8}{17}, \sin \beta = \frac{15}{17}$ với $0 < \alpha < \frac{\pi}{3}, 0 < \beta < \frac{\pi}{2}$. Chứng minh rằng $\alpha + \beta = \frac{\pi}{2}$

Gợi ý làm bài

Ta có:

$$\begin{aligned} \cos \alpha &= \sqrt{1 - \frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}; \\ \cos \beta &= \sqrt{1 - \frac{225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17} \end{aligned}$$

Do đó:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\frac{8}{17} \cdot \frac{8}{17} + \frac{15}{17} \cdot \frac{15}{17} = \frac{289}{289} = 1$$

Vì $0 < \alpha < \frac{\pi}{3}, 0 < \beta < \frac{\pi}{2}$ nên từ đó suy ra $\alpha + \beta = \frac{\pi}{2}$

Bài 18 trang 193 Sách bài tập (SBT) Toán Đại số 10



Không dùng bảng số và máy tính, chứng minh rằng

a) $\sin 20^0 + 2 \sin 40^0 - \sin 100^0 = \sin 40^0$

b) $\frac{\sin(45^0 + \alpha) - \cos(45^0 + \alpha)}{\sin(45^0 + \alpha) + \cos(45^0 + \alpha)} = \tan \alpha$

c) $\frac{3\cot^2 15^0 - 1}{3 - \cot^2 15^0} = -\cot 15^0$

d) $\sin 200^0 \sin 310^0 + \cos 340^0 \cos 50^0 = \frac{\sqrt{3}}{2}$

Gợi ý làm bài

a)

$$\begin{aligned} & \sin 20^0 + 2 \sin 40^0 - \sin 100^0 \\ &= (\sin 20^0 - \sin 100^0) + 2 \sin 40^0 \\ &= 2 \cos 60^0 \sin(-40^0) + 2 \sin 40^0 \\ &= -\sin 40^0 + 2 \sin 40^0 = \sin 40^0 \end{aligned}$$

b)

$$\begin{aligned} & \frac{\sin(45^0 + \alpha) - \cos(45^0 + \alpha)}{\sin(45^0 + \alpha) + \cos(45^0 + \alpha)} \\ &= \frac{\sin(45^0 + \alpha) - \sin(45^0 - \alpha)}{\sin(45^0 + \alpha) + \sin(45^0 - \alpha)} \\ &= \frac{2 \cos 45^0 \sin \alpha}{2 \sin 45^0 \cos \alpha} = \frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} = \tan \alpha \end{aligned}$$

c)

$$\begin{aligned} & \frac{3\cot^2 15^0 - 1}{3 - \cot^2 15^0} = \frac{\cot^2 30^0 \cot^2 15^0 - 1}{\cot^2 30^0 - \cot^2 15^0} \\ &= \frac{\cot 30^0 \cot 15^0 + 1}{\cot 30^0 - \cot 15^0} \cdot \frac{\cot 30^0 \cot 15^0 - 1}{\cot 30^0 + \cot 15^0} \end{aligned}$$

Mặt khác ta có



$$\cot(\alpha + \beta) = \frac{\cos(\alpha + \beta)}{\sin(\alpha + \beta)} = \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}$$

Chia cả tử và mẫu của biểu thức cho $\sin \alpha \sin \beta$ ta được

$$\cot(\alpha + \beta) = \frac{\cot \alpha \cot \beta - 1}{\cot \alpha + \cot \beta}$$

Tương tự

$$\cot(\alpha - \beta) = \frac{\cot \alpha \cot \beta + 1}{\cot \beta - \cot \alpha}$$

Do đó

$$A = \cot(15^\circ - 30^\circ) \cot(15^\circ + 30^\circ) = -\cot 15^\circ$$

d)

$$\begin{aligned} & \sin 200^\circ \sin 310^\circ + \cos 340^\circ \cos 50^\circ \\ &= \sin(180^\circ + 20^\circ) \sin(360^\circ - 50^\circ) + \cos(360^\circ - 20^\circ) \cos 50^\circ \\ &= (-\sin 20^\circ)(-\sin 50^\circ) + \cos 20^\circ \cos 50^\circ \\ &= \cos 50^\circ \cos 20^\circ + \sin 50^\circ \sin 20^\circ \\ &= \cos(50^\circ - 20^\circ) = \frac{\sqrt{3}}{2} \end{aligned}$$

Bài 19 trang 194 Sách bài tập (SBT) Toán Đại số 10

Chứng minh rằng các biểu thức sau là những hằng số không phụ thuộc α, β

a) $\sin 6\alpha \cot 3\alpha - \cos 6\alpha$

b) $[\tan(90^\circ - \alpha) - \cot(90^\circ + \alpha)]^2 - [\cot(180^\circ + \alpha) + \cot(270^\circ + \alpha)]^2$

c) $(\tan \alpha - \tan \beta) \cot(\alpha - \beta) - \tan \alpha \tan \beta$

d) $\cot \frac{\alpha}{3} - \tan \frac{\alpha}{3} \tan \frac{2\alpha}{3}$

Gợi ý làm bài

a)



$$\sin 6\alpha \cot 3\alpha - \cos 6\alpha$$

$$= 2 \sin 3\alpha \cos 3\alpha \cdot \frac{\cos 3\alpha}{\sin 3\alpha} - (2\cos^2 3\alpha - 1)$$

$$= 2\cos^2 3\alpha - 2\cos^2 3\alpha + 1 = 1$$

b)

$$[\tan(90^\circ - \alpha) - \cot(90^\circ + \alpha)]^2 - [\cot(180^\circ + \alpha) + \cot(270^\circ + \alpha)]^2$$

$$= (\cot \alpha + \tan \alpha)^2 - (\cot \alpha - \tan \alpha)^2$$

$$= \cot^2 \alpha + 2 + \tan^2 \alpha - \cot^2 \alpha + 2 - \tan^2 \alpha = 4$$

c)

$$(\tan \alpha - \tan \beta) \cot(\alpha - \beta) - \tan \alpha \tan \beta$$

$$= \frac{\tan \alpha - \tan \beta}{\tan(\alpha - \beta)} - \tan \alpha \tan \beta$$

$$= 1 + \tan \alpha \tan \beta - \tan \alpha \tan \beta = 1$$

d)

$$\begin{aligned} & (\cot \frac{\alpha}{3} - \tan \frac{\alpha}{3}) \tan \frac{2\alpha}{3} \\ &= \left(\frac{\cos \frac{\alpha}{3}}{\sin \frac{\alpha}{3}} - \frac{\sin \frac{\alpha}{3}}{\cos \frac{\alpha}{3}} \right) \frac{\sin \frac{2\alpha}{3}}{\cos \frac{2\alpha}{3}} \\ &= \frac{\cos^2 \frac{\alpha}{3} - \sin^2 \frac{\alpha}{3}}{\sin \frac{\alpha}{3} \cos \frac{\alpha}{3}} \cdot \frac{\sin \frac{2\alpha}{3}}{\cos \frac{2\alpha}{3}} \\ &= \frac{\cos \frac{2\alpha}{3}}{\frac{1}{2} \sin \frac{2\alpha}{3}} \cdot \frac{\sin \frac{2\alpha}{3}}{\cos \frac{2\alpha}{3}} = 2 \end{aligned}$$

Bài 20 trang 194 Sách bài tập (SBT) Toán Đại số 10

Không sử dụng bảng số và máy tính, hãy tính

a) $\sin^4 \frac{\pi}{16} + \sin^4 \frac{3\pi}{16} + \sin^4 \frac{5\pi}{16} + \sin^4 \frac{7\pi}{16}$

b) $\cot 7,5^\circ + \tan 67,5^\circ - \tan 7,5^\circ - \cot 67,5^\circ$

Gợi ý làm bài

$$\begin{aligned}
 \text{a) } & \sin^4 \frac{\pi}{16} + \sin^4 \frac{3\pi}{16} + \sin^4 \frac{5\pi}{16} + \sin^4 \frac{7\pi}{16} \\
 &= \left(\frac{1 - \cos \frac{\pi}{8}}{2} \right)^2 + \left(\frac{1 - \cos \frac{3\pi}{8}}{2} \right)^2 + \left(\frac{1 - \cos \frac{5\pi}{8}}{2} \right)^2 + \left(\frac{1 - \cos \frac{7\pi}{8}}{2} \right)^2 \\
 &= \frac{1}{4} \left(1 - 2 \cos \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + 1 - 2 \cos \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8} + 1 - 2 \cos \frac{5\pi}{8} + \cos^2 \frac{5\pi}{8} + 1 - 2 \cos \frac{7\pi}{8} + \cos^2 \frac{7\pi}{8} \right) \\
 &= 1 - \frac{1}{2} \left(\cos \frac{\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{5\pi}{8} + \cos \frac{7\pi}{8} \right) + \frac{1}{4} \left(\frac{1 + \cos \frac{\pi}{4}}{2} + \frac{1 + \cos \frac{3\pi}{4}}{2} + \frac{1 + \cos \frac{5\pi}{4}}{2} + \frac{1 + \cos \frac{7\pi}{4}}{2} \right) \\
 &= 1 - \frac{1}{2} \left(\cos \frac{\pi}{8} + \cos \frac{3\pi}{8} - \cos \frac{3\pi}{8} - \cos \frac{\pi}{8} \right) + \frac{1}{8} \left(4 + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) \\
 &= \frac{3}{2} \\
 \text{b) } & \cot 7,5^\circ + \tan 67,5^\circ - \tan 7,5^\circ - \cot 67,5^\circ \\
 &= \frac{\cos 7,5^\circ}{\sin 7,5^\circ} - \frac{\sin 7,5^\circ}{\cos 7,5^\circ} + \frac{\sin 67,5^\circ}{\cos 67,5^\circ} - \frac{\cos 67,5^\circ}{\sin 67,5^\circ} \\
 &= \frac{\cos^2 7,5^\circ - \sin^2 7,5^\circ}{\sin 7,5^\circ \cos 7,5^\circ} + \frac{\sin^2 67,5^\circ - \cos^2 67,5^\circ}{\sin 67,5^\circ \cos 67,5^\circ} \\
 &= \frac{\cos 15^\circ}{\frac{1}{2} \sin 15^\circ} - \frac{\cos 135^\circ}{\frac{1}{2} \sin 135^\circ} \\
 &= \frac{2(\sin 135^\circ \cos 15^\circ - \cos 135^\circ \sin 15^\circ)}{\sin 15^\circ \sin 135^\circ} \\
 &= \frac{\sin(135^\circ - 15^\circ)}{\sin(45^\circ - 30^\circ) \sin(180^\circ - 45^\circ)} \\
 &= \frac{2 \sin 120^\circ}{(\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ) \sin 45^\circ} \\
 &= \frac{\sqrt{3}}{\frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) \cdot \frac{\sqrt{2}}{2}} \\
 &= \frac{4\sqrt{3}}{\sqrt{3} - 1} = 6 + 2\sqrt{3}
 \end{aligned}$$

Bài 21 trang 194 Sách bài tập (SBT) Toán Đại số 10

Rút gọn các biểu thức

$$\text{a) } \frac{\sin 2\alpha + \sin \alpha}{1 + \cos 2\alpha + \cos \alpha}$$

$$\text{b) } \frac{4\sin^2 \alpha}{1 - \cos^2 \frac{\alpha}{2}}$$

$$\text{c) } \frac{1 + \cos \alpha - \sin \alpha}{1 - \cos \alpha - \sin \alpha}$$

$$\frac{1 + \sin \alpha - 2\sin^2(45^\circ - \frac{\alpha}{2})}{4\cos \frac{\alpha}{2}}$$

d)

Gợi ý làm bài

$$\begin{aligned} \text{a) } \frac{\sin 2\alpha + \sin \alpha}{1 + \cos 2\alpha + \cos \alpha} &= \frac{\sin \alpha(2 \cos \alpha + 1)}{2\cos^2 \alpha + \cos \alpha} \\ &= \frac{\sin \alpha(2 \cos \alpha + 1)}{\cos \alpha(2\cos \alpha + 1)} = \tan \alpha \end{aligned}$$

$$\text{b) } \frac{4\sin^2 \alpha}{1 - \cos^2 \frac{\alpha}{2}} = \frac{16\sin^2 \frac{\alpha}{2} \cos^2 \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2}} = 16\cos^2 \frac{\alpha}{2}$$

$$\begin{aligned} \text{c) } \frac{1 + \cos \alpha - \sin \alpha}{1 - \cos \alpha - \sin \alpha} &= \frac{2\cos^2 \frac{\alpha}{2} - 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2\sin^2 \frac{\alpha}{2} - 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} \\ &= \frac{2\cos \frac{\alpha}{2}(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2})}{2\sin \frac{\alpha}{2}(\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2})} = -\cot \frac{\alpha}{2} \end{aligned}$$

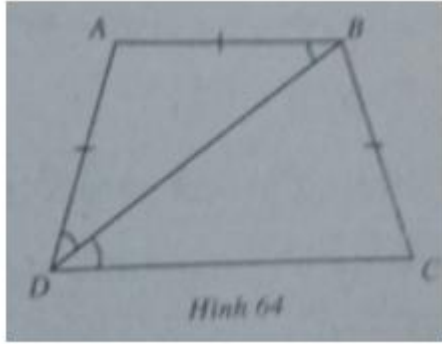
$$\begin{aligned} \text{d) } \frac{1 + \sin \alpha - 2\sin^2(45^\circ - \frac{\alpha}{2})}{4\cos \frac{\alpha}{2}} &= \frac{\sin \alpha + \cos(90^\circ - \alpha)}{4\cos \frac{\alpha}{2}} \\ &= \frac{\sin \alpha + \sin \alpha}{4\cos \frac{\alpha}{2}} = \frac{4\sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{4\cos \frac{\alpha}{2}} = \sin \frac{\alpha}{2} \end{aligned}$$

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Cho hình thang cân ABCD có đáy nhỏ AB = AD. Biết $\tan \widehat{BDC} = \frac{3}{4}$ tính các giá trị lượng giác của $\widehat{BAD}$

Gợi ý làm bài

Ta có (h.64)



$$\widehat{ABD} = \widehat{ADB}$$

$$\widehat{ABD} = \widehat{BDC}$$

$$\Rightarrow \widehat{BDC} = \widehat{ADB}$$

$$\text{Suy ra } \widehat{BAD} = \pi - 2\widehat{BDC}$$

Từ đó ta có:

$$\begin{aligned}\tan \widehat{BAD} &= -\tan 2\widehat{BDC} = -\frac{2 \tan \widehat{BDC}}{1 - \tan^2 \widehat{BDC}} \\ &= -\frac{2 \cdot \frac{3}{4}}{1 - \frac{9}{16}} = -\frac{3}{2} \cdot \frac{16}{7} = -\frac{24}{7}\end{aligned}$$

Vì $\frac{\pi}{2} < \widehat{BAD} < \pi$ nên $\cos \widehat{BAD} < 0$. Do đó

$$\begin{aligned}\cos \widehat{BAD} &= -\frac{1}{\sqrt{1 + \tan^2 \widehat{BAD}}} \\ &= -\frac{1}{\sqrt{1 + \frac{576}{49}}} = -\frac{7}{25}\end{aligned}$$

$$\begin{aligned}\sin \widehat{BAD} &= \cos \widehat{BAD} \cdot \tan \widehat{BAD} \\ &= \frac{-7}{25} \cdot \frac{-24}{7} = \frac{24}{25}\end{aligned}$$

Xem thêm các bài tiếp theo tại: <https://vndoc.com/giai-toan-lop-10>